Edge-Preserving Image Decomposition using L1 Fidelity with L0 Gradient

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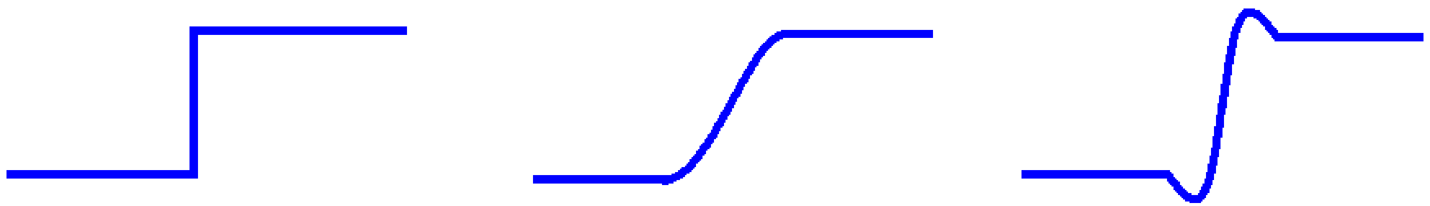
SIGGRAPH ASIA 2012

Image Decomposition

- Image decomposition : a tool for image editing.
- The intensity can be decomposed into base layer and detail layer.
- The basic example is un-sharp masking.



Input

$$L(x,y) = B(x,y) + D(x,y)$$

Base layer

$$B(x,y)$$

Output

$$L_{EN}(x,y) = B(x,y) + k \cdot D(x,y)$$

Image Decomposition

- **Pyramid**: [Burt & Adelson 83]
- **Retinex**: [Jobson et al. 97]
- **EMD**: [Huang 98]
- **Bilateral Filtering**: [Tomasi & Manduchi 98]
- **Fast Bilateral Filtering**: [Durand & Dorsey 02]
- **Weighted Least Square**: [Farbman et al.08]
- **Local Extrema**: [Subr et al. 09]
- **Guided Filtering**: [He et al. 10]
- **L0 Gradient Minimization**: [Xu et al. 11]

Edge-Preserving Image Decomposition

- **Weighted Least Square: [Farbman et al.08]**

$$\min_{B(x,y)} \sum_{(x,y)} |B(x,y)-L(x,y)|^2 + \lambda \cdot \left(b_x(x,y)\cdot\left(\frac{\partial B(x,y)}{\partial x}\right)^2 + b_y(x,y)\cdot\left(\frac{\partial B(x,y)}{\partial y}\right)^2\right)$$

- **L0 Gradient Minimization: [Xu et al. 11]**

$$\min_{B(x,y)} \sum_{(x,y)} |B(x,y)-L(x,y)|^2 + \lambda \cdot C(x,y) \qquad C(x,y) = \left\{ \# \left| \left|\frac{\partial B(x,y)}{\partial x}\right| + \left|\frac{\partial B(x,y)}{\partial y}\right| \neq 0 \right. \right\}$$

- **We decompose the signal using L1 fidelity with L0 gradient. (sparsity measurement)**

$$\min_{B(x,y)} \sum_{(x,y)} |B(x,y)-L(x,y)| + \lambda \cdot C(x,y) \qquad C(x,y) = \left\{ \# \left| \left|\frac{\partial B(x,y)}{\partial x}\right| + \left|\frac{\partial B(x,y)}{\partial y}\right| \neq 0 \right. \right\}$$

Image Smoothing

- **The input signal can be decomposed into base layer and detail layer:**

$$L(x,y) = B(x,y) + D(x,y)$$

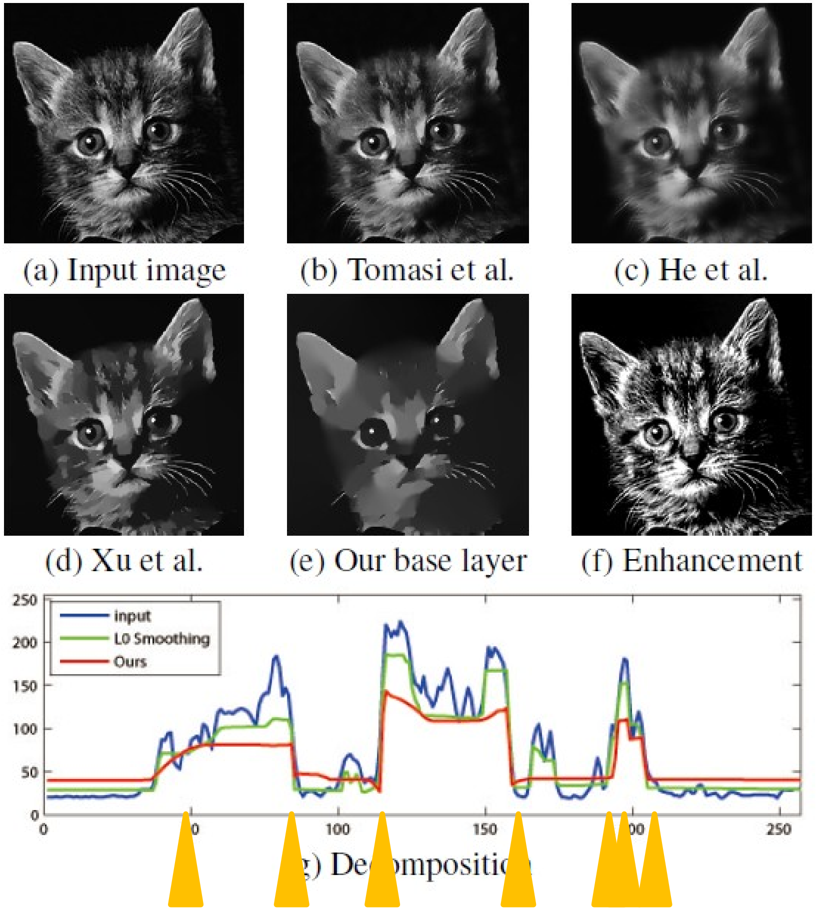
- **using minimization:**

$$\min_{B(x,y)} \sum_{(x,y)} | B(x,y) - L(x,y) | + \lambda \cdot C(x,y)$$

- **Approximate L1 fidelity by using** $w(x,y)$ **:**

$$\min_{B,w} \alpha \cdot \sum_{(x,y)} | B(x,y) - L(x,y) - w(x,y) |^2 + \sum_{(x,y)} | w(x,y) | + \lambda \cdot C(x,y)$$

Error between B(x,y)-L(x,y) and w(x,y)   Approximation



(g) Decomposition

Image Smoothing

- **Approximate L0 gradient using** $C(a_x(x,y),a_y(x,y))$

$$\min_{B,w,a_x,a_y} \alpha \cdot \sum_{(x,y)} |B(x,y)-L(x,y)-w(x,y)|^2 + \sum_{(x,y)} |w(x,y)| + \lambda \cdot C(a_x(x,y),a_y(x,y))$$

$$+\beta \cdot \left(\left|\frac{\partial B(x,y)}{\partial x} - a_x(x,y)\right|^2 + \left|\frac{\partial B(x,y)}{\partial y} - a_y(x,y)\right|^2\right) \qquad \text{(Equation *)}$$

where the L0 norm of gradient (L0 gradient):

$$C(a_x(x,y),a_y(x,y)) = \#\left\{(x,y) \,\middle|\, \left|\frac{\partial B(x,y)}{\partial x}\right| + \left|\frac{\partial B(x,y)}{\partial y}\right| \neq 0\right\}$$

Solver: Computing B(x,y)

- **Since $B(x,y)$ is independent to $w(x,y)$, $a_x(x,y)$ and $a_y(x,y)$,we can solve the equation* by using alternating minimization:**

$$\min_B \frac{\alpha}{\beta} \cdot \sum_{(x,y)} | B(x,y) - L(x,y) - w(x,y) |^2$$

$$+ \left| \frac{\partial B(x,y)}{\partial x} - a_x(x,y) \right|^2 + \left| \frac{\partial B(x,y)}{\partial y} - a_y(x,y) \right|^2$$

- **Using FFT to speed up the whole process:**

$$B(x,y) = F^{-1}\left\{ \frac{\alpha F(L) + \alpha F(w) + \beta F^*(\partial x) F(a_x) + \beta F^*(\partial y) F(a_y)}{\alpha + \beta F^*(\partial x) F(\partial x) + \beta F^*(\partial y) F(\partial y)} \right\}$$

Solver: Computing w(x,y), a_x(x,y) & a_y(x,y)

- **Moreover, $w(x,y)$ is also independent to $a_x(x,y)$ and $a_y(x,y)$, we can solve $w(x,y)$ first:**

$$\min_{w(x,y)} |w(x,y)| + \alpha \cdot \sum_{(x,y)} |B(x,y) - L(x,y) - w(x,y)|^2$$

$$w(x,y) = \max\left\{ |B(x,y) - L(x,y)| - \frac{1}{2\alpha}, 0 \right\} \frac{B(x,y) - L(x,y)}{|B(x,y) - L(x,y)|}$$

Solver: Computing w(x,y), a_x(x,y) & a_y(x,y)

- **Using a binary map $H(|a_x(x,y)|+|a_y(x,y)|)$, we can obtain the L0 norm of gradient $C(a_x(x,y),a_y(x,y))$:**

$$\min_{a_x,a_y} \frac{\lambda}{\beta}\cdot C(a_x(x,y),a_y(x,y)) + \left|\frac{\partial B(x,y)}{\partial x} - a_x(x,y)\right|^2 + \left|\frac{\partial B(x,y)}{\partial y} - a_y(x,y)\right|^2$$

$$C(a_x(x,y),a_y(x,y)) = H(|a_x(x,y)|+|a_y(x,y)|) = \begin{cases} 1, |a_x(x,y)|+|a_y(x,y)| \neq 0 \\ 0, |a_x(x,y)|+|a_y(x,y)| = 0 \end{cases}$$

General Form for Enhancement

- **Since we obtain the final base layer $\hat{B}(x,y)$, we can also obtain our corresponding detail layer $\hat{D}(x,y)$:**

$$\hat{D}(x,y) = L(x,y) - \hat{B}(x,y)$$

- **The general form for image enhancement:**

$$L_{EN}(x,y) = s \cdot W \cdot \left(\frac{\hat{B}(x,y)}{W}\right)^{\gamma} + G \cdot \hat{D}(x,y)$$

s **is the scale parameter,** W **is the white value,**

γ **is the parameter for gamma correction, and**

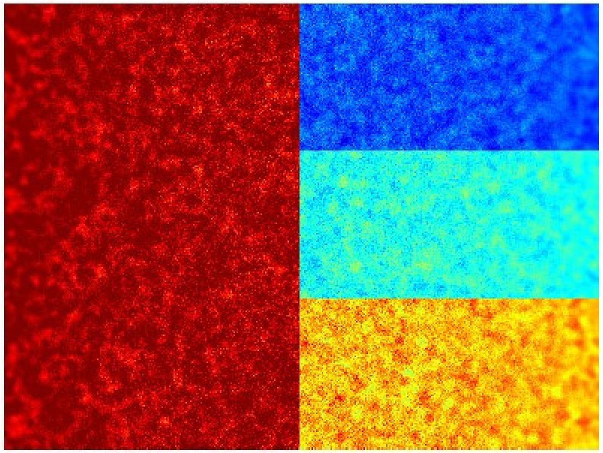
G **is the gain for detail boosting.**

Image Smoothing

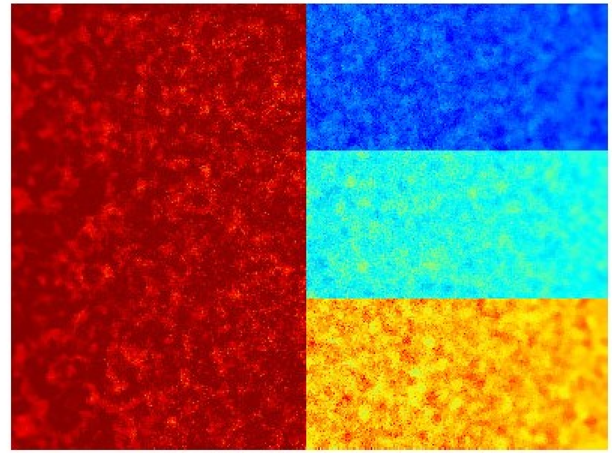
Input

Ours

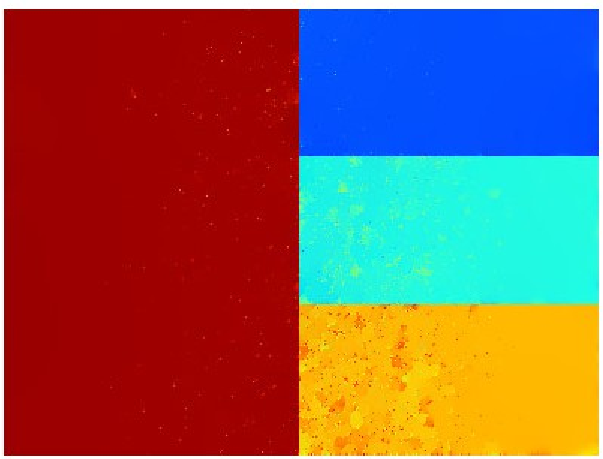
Noise Removal



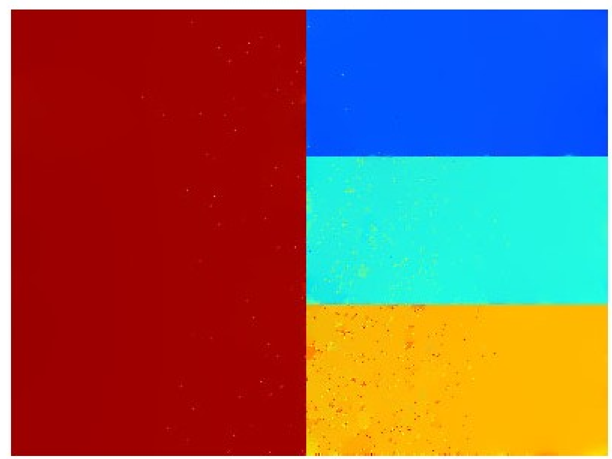
Noisy Input



WLS, $|\ldots|^2 + \lambda \cdot (\ldots)^2$



L0 Smooting, $|\ldots|^2 + \lambda \cdot \|\ldots\|_0$

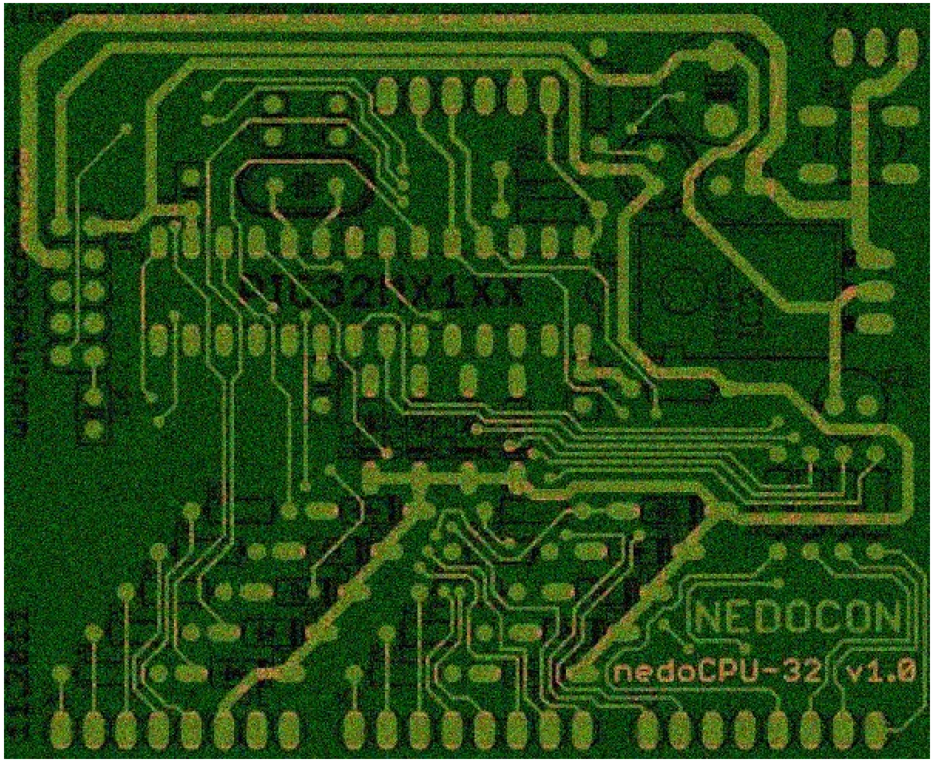


Ours, $|\ldots| + \lambda \cdot \|\ldots\|_0$

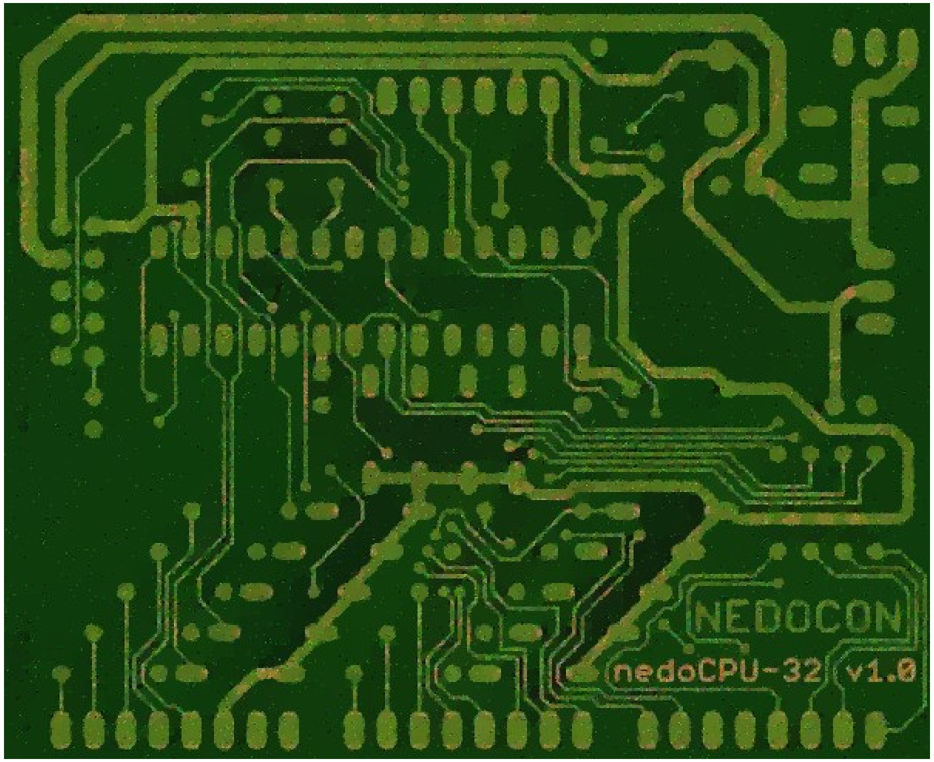
Using the same balancing parameter

$$\lambda = 0.1$$

Noise Removal



Noisy Input



Ours

Artifact Removal

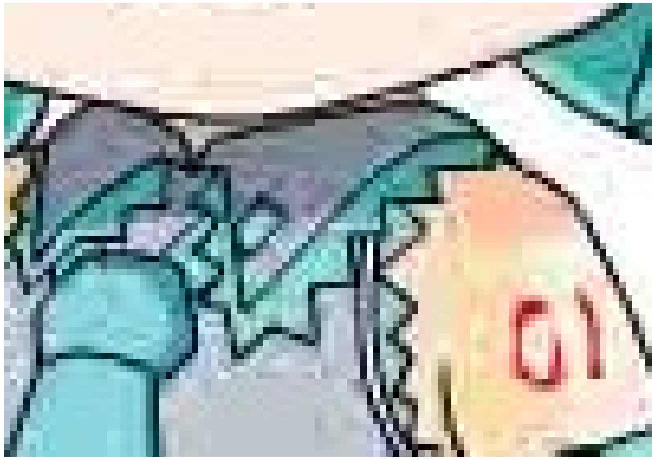

Non-Degraded BMP


Degraded JPG


Mean Shift


Ours

Artifact Removal

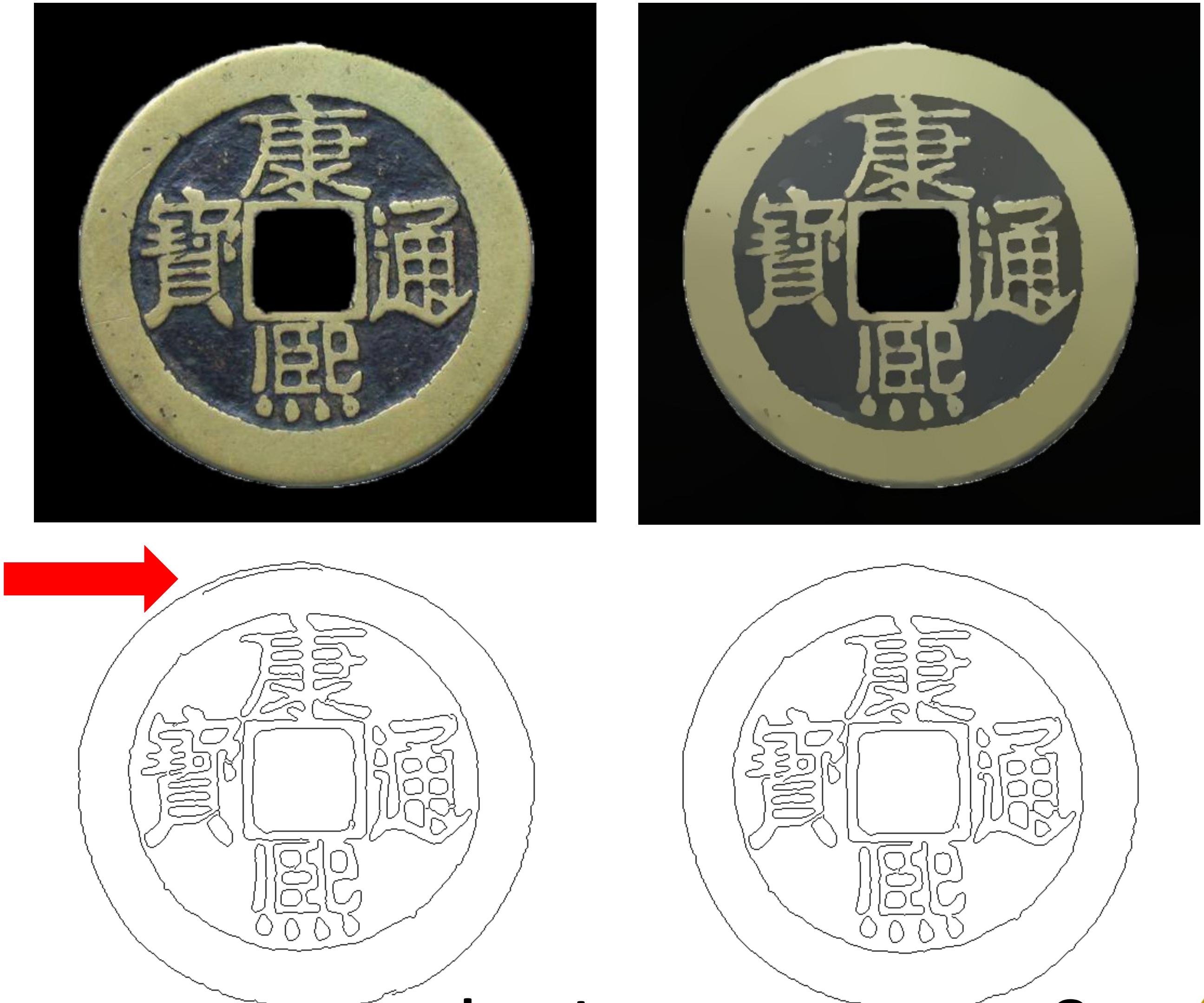


Degraded JPG

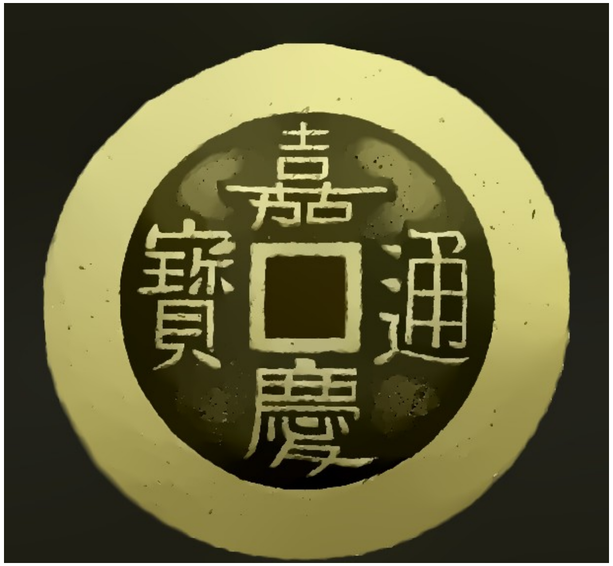
Artifact Removal

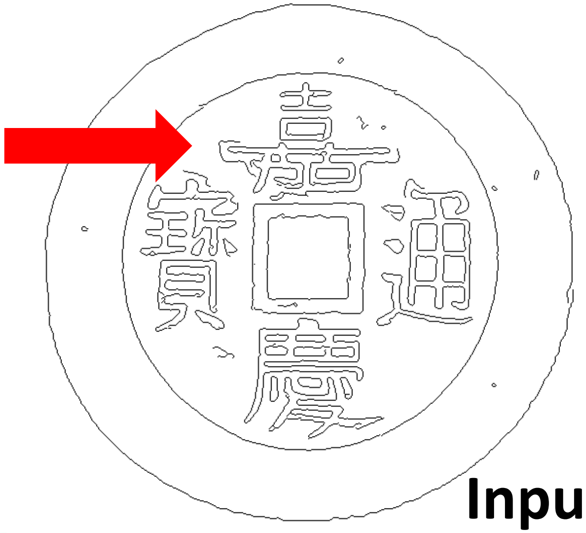
Ours

Edge Detection



Edge Detection

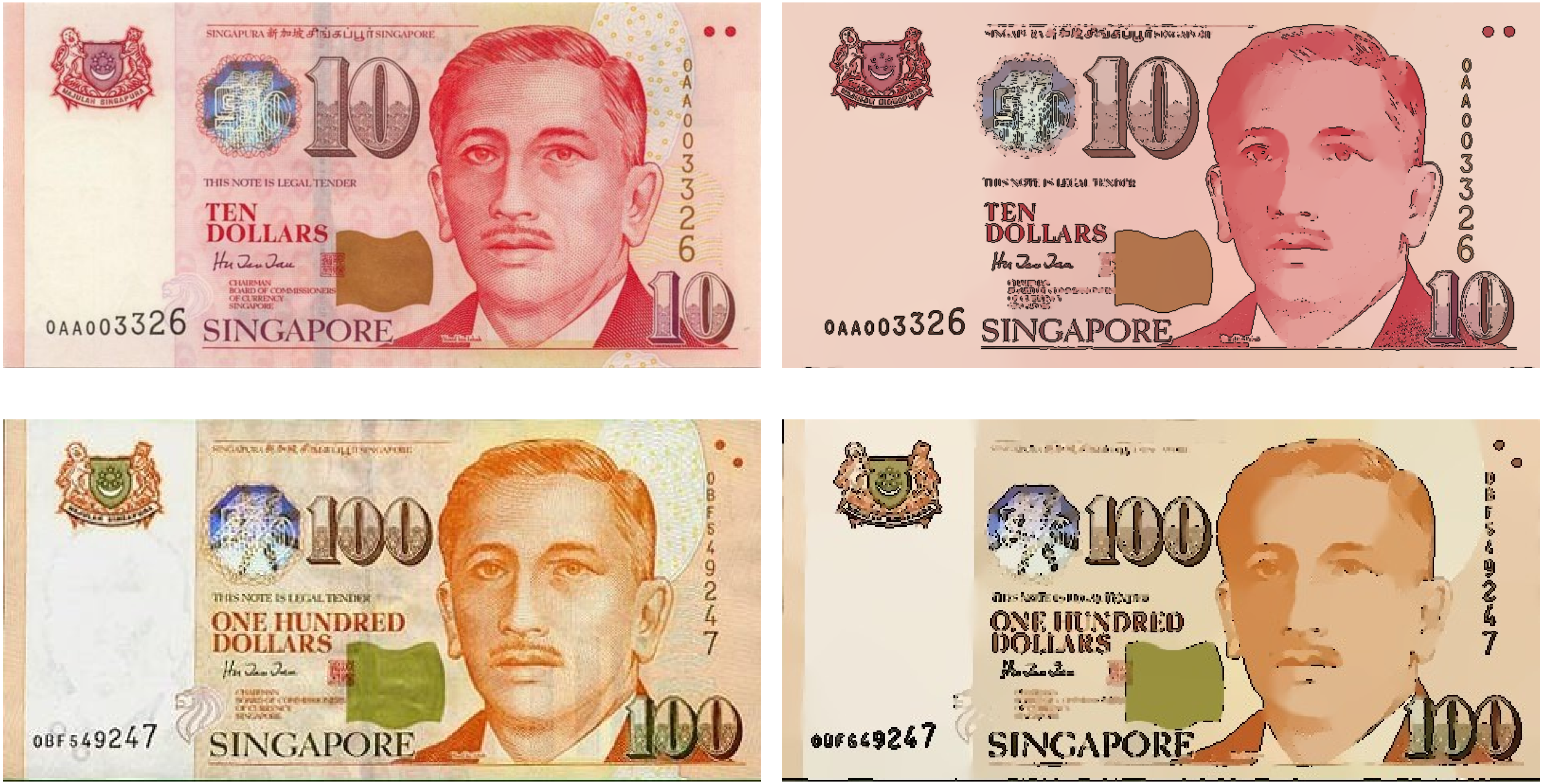




Input

Ours

Stylization



Input Ours

Stylization

Input

Ours

Image Enhancement

Input

Ours

Image Enhancement

Input

Ours

Low Backlight

Original, 100%

Original, 40%

Backlight:
Scale down to 40%

Our, 40%

Backlight:
Scale down to 40%

Image:
Compensate the brightness
Boost the detail parts

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